Abstracts

"From martingales to dynamical systems"

October 8,9 and 10 2014

Speaker: Jean-René Chazottes, Ecole Polytechnique Paris

Title: Concentration inequalities for dynamical systems

Abstract: In the context of discrete-time dynamical systems, a concentration inequality is a bound for the probability that an observation $K(x, Tx, , T^n x)$ deviates from its expectation by a given amount. Here T is a map acting on a space X and we consider special probability measures invariant under T. The point is that such bounds involve a constant depending only on T (not in n in particular) and the sum of the squared Lipschitz constants of K. As we shall see, such inequalities are based on classical martingale inequalities.

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Speaker: Christophe Cuny, Ecole centrale Paris

Title: Limit theorems by mean of the forward-backward martingale decomposition.

Abstract: Let P be an ergodic (and stationary) Markov operator on $L^2(m)$ and $(W_n)_{n\in\mathbb{Z}}$ be the corresponding (canonical) Markov chain. Liming Wu (1999) and Stefano Olla (2001) considered the (Hilbert) space $\mathcal{H}_{-1} := \sqrt{I - P_s}L^2(m)$, where $P_s := (P + P^*)/2$, and proved that for $f \in \mathcal{H}_{-1}$, we have a decomposition: $f(W_0) = D^+f + D^-f + Rf - Rf\circ\theta$, where θ stands for the shift, D^+f is a martingale difference in L^2 and D^-f is a difference of reversed martingale. This forward-backward martingale decomposition allows one to derive tightness type results for $S_n = f(W_0) + \cdots + f(W_{n-1})$. In particular one then obtains several limit theorems whenever $f \in \mathcal{B} := \overline{(I - P)\mathcal{H}_{-1}}^{\mathcal{H}_{-1}}$. Olla and Wu also gave a sufficient condition for the identity $\mathcal{B} = \mathcal{H}_{-1}$. We will see that this condition may be interpreted in terms of the so called numerical range and give examples of operators satisfying that condition and for which $\mathcal{H}_{-1} = \{f \in L^2(m) : \sum_{n\geq 1} \|P^n f\|_{L^2(m)} < \infty\}$.

This talk is based on a joint work with Michael Lin.

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Speaker: Jérôme Dedecker, Université Paris Descartes, France.

Title: Central limit theorem for the Wasserstein distance between the empirical distribution and the invariant distribution of a stationary sequence.

Abstract: We adapt the martingale approximation result by Gordin (1973) to the case of random variables with values in the Banach space $L_1(dt)$. As a consequence, we obtain a new criterion for the weak convergence of the Wasserstein distance between the empirical distribution and the invariant distribution. In the α-dependent case, the condition is weaker than the condition obtained from the general criterion given in Cuny (2014). In particular the Wasserstein empirical CLT holds for bounded Harris recurrent Markov chains as soon as the mixing coefficients are summable, which is the same condition as for the usual CLT.

This talk is based on a joint work with F. Merlevède.

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Speaker: Bernard Delyon, Université de Rennes, France.

Title: Exponential inequalities for dependent processes.

Abstract: TBA

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Speaker: Christophe Dupont, Université de Rennes, France.

Title: Applications of the Central Limit Theorem in holomorphic dynamics.

Abstract: We give two applications of the CLT in holomorphic dynamics. The first one concerns rational maps acting on the Riemann sphere : if μ denotes the measure of maximal entropy then a gap occurs between the borelian support of μ and the topological support of mu (if the map is generic), this is due to Zdunik. The second application is a direct proof of the absolute continuity of μ when Pesin's formula is satisfied, the argument also works in several complex variables. In both cases the observable is the logarithm of the jacobian of the map. Remarkably the CLT is available for this function even if it is unbounded near the critical points.

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Speaker: Sébastien Gouëzel, Université de Rennes, France.

Title: Entropy of random walks on hyperbolic groups.

Abstract: TBA

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Speaker: Ion Grama, Université de Bretagne-Sud, France.

Title: Conditional limit theorems for products of random matrices

Abstract: Consider the product $G_n = g_n...g_1$ of the random matrices $g_1, ..., g_n$ in $GL(d, \mathbb{R})$ and the random process $G_n v = g_n...g_1 v$ in $\mathbb{R}^d$ starting at point $v \in \mathbb{R}^d \setminus \{0\}$. It is well known that under appropriate assumptions, the sequence $(\log \|G_n v\|)_{n \geq 1}$ behaves like a sum of i.i.d. r.v.'s and satisfies standard classical properties such as the law of large numbers, law of iterated logarithm and the central limit theorem. Denote by $\mathbb{B}$ the closed unit ball in $\mathbb{R}^d$ and by $\mathbb{B}^c$ its complement. For any $v \in \mathbb{B}^c$ define the exit time of the random process $G_n v$ from $\mathbb{B}^c$ by $\tau_v = \min\{n \geq 1 : G_n v \in \mathbb{B}\}$. We establish the asymptotic as $n \to \infty$ of the probability of the event $\{\tau_v > n\}$ and find the limit law for the quantity $\frac{1}{\sqrt{n}} \log \|G_n v\|$ conditioned that $\tau_v > n$. Our proofs rely upon a strong approximation result for Markov chains.

The talk is based on a joint work with Emile Le Page and Marc Peigné.

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Speaker: Mike Hochman, The Hebrew University, Israël.

Title: Equidistribution of random points

Abstract: Suppose you are given a probability measure μ on a compact metric space X, and are interested in the orbital distribution of μ-typical points (for example, an SRB measure is one which attracts typical points of the volume measure). I will discuss a lemma that relates these orbital distributions to the fine-scale structure of the measure (martingales will appear). As time permits I will discuss applications to the existence of normal numbers in fractals. The talk is based on joint work with Pablo Shmerkin.

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Speaker: Zemer Kosloff, University of Warwick, England.

Title: Symmetric Birkhoff sums in infinite ergodic theory: Failure of the ergodic theorem and peculiar behaviour.

Abstract: TBA

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Speaker: Lingmin Liao, Université Paris-Est Créteil, France.

Title: Minimality of rational maps with good reduction on the projective line of the field of p-adic numbers

Abstract: Rational maps having good reduction in the field $\mathbb{Q}_p$ of p-adic numbers are studied as dynamical systems on the projective line $\mathbb{P}^1(\mathbb{Q}_p)$ of $\mathbb{Q}_p$. For general prime p, a criterion of the minimality of such a rational map is obtained. For the case $p = 2$, a complete characterization of minimal rational maps with good reduction is given in

terms of their coefficients. It is also proved that a rational map of degree 2 or 3 can never be minimal on the whole space $\mathbb{P}^1(\mathbb{Q}_p)$. This is a joint work with Ai-Hua Fan, Shi-Lei Fan and Yue-Fei Wang.

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Speaker: Quansheng Liu, Université de Bretagne-Sud, France.

Title: Cramér's large deviation expansion for martingales

Abstract: We present Cramér's large deviation expansion for martingales under Bernstein's condition or Cramér's condition. The range and the order of the relative error of our expansion are the same as in Cramér's expansion for sums of independent and identically distributed random variables. The new expansion formula extends the range of the known ones obtained for bounded martingale differences, while keeping the same order for the remainder term. As a corollary, it implies a moderate deviation principle. The approach is based on Cramér's method of change of probability developed by Grama and Haeusler for martingales. This talk is based on a joint work with Xiequan Fan and Ion Grama.

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Speaker: Marc Peigné, Université François Rabelais, France.

Title: Counting in negative curvature and infinite measure.

Abstract: Let X be a Riemannian manifold with negative curvature $-b^2 \leq K \leq -a^2 < 0$, with fundamental group Γ. We investigate here the asymptotic behavior of the *counting function* N_Γ of Γ defined by

$$N_\Gamma(x, y, R) := \text{card}\{\gamma \in \Gamma : d(x, \gamma \cdot y) \leq R\}$$

where d denotes the induced distance on the universal covering $\tilde{X}$ of X and $x, y \in \tilde{X}$.

Since the geodesic flow $(g_t)_t$ on T^1M is hyperbolic, it possesses a lot of invariant measures; one of them, the so-called *Bowen-Margulis measure* μ_{BM}, has been intensively studied in the last decades.

When μ_{BM} is finite, the function N_Γ is purely exponential at infinity: there exists constants $C = C(x, y) > 0$ such that $N_\Gamma(x, y, R) \sim Ce^{\delta_\Gamma R}$ as $R \to +\infty$.

When $\mu_{BM}(T^1M) = +\infty$, one only knows that $N_\Gamma(x, y, R) = o\Big(e^{\delta_\Gamma R}\Big)$. We investigate this last case for a particular class of groups Γ for which the fact that $\mu_{BM}(T^1M) = +\infty$ may be reduced to the divergence of some series connected to the geometry of ends of the manifold X. This study relies on some renewal theorem for Markovian random walks with infinite means whose increments belong to the domain of attraction of stable laws.

This talk is based on a joint work with F. Dal'bo, J.C. Picaud & A. Sambusetti, and a work in progress by P. Vidotto.

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Speaker: Françoise Pène, Université de Bretagne Occidentale, France.

Title: Rates in the strong invariance principle for functionals of ergodic automorphisms of the torus.

Abstract: We consider an ergodic automorphism T of the torus endowed with the Lebesgue measure and a centered real-valued observable f. We are interested in the stochastic properties of the Birkhoff sum $f \circ T + ... + f \circ T^n$, namely in the rate of convergence in the almost sure invariance principle. Our results apply under weak assumptions on the continuity modulus of f or on its Fourier coefficients. Our approach is based on general results for stationary sequences, the proofs of which use martingales approximations.

This talk is based on a joint work with J. Dedecker and F. Merlevède.

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Speaker: Benoît Saussol, Université de Bretagne Occidentale, France.

Title: Poisson law for some nonuniformly hyperbolic dynamical systems with polynomial rate of mixing.

Abstract: We consider some nonuniformly hyperbolic invertible dynamical systems which are modeled by a Gibbs-Markov-Young tower. We assume a polynomial tail for the inducing time and a polynomial control of hyperbolicity, as introduced by Alves, Pinheiro and Azevedo. These systems admit a physical measure with polynomial rate of mixing. In this paper we prove that the distribution of the number of visits to a ball $B(x, r)$ converges to a Poisson distribution as the radius $r \to 0$ and after suitable normalization.

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Speaker: Stéphane Seuret, Université Paris-Est Créteil, France.

Title: Lacunary Fourier series and continued fractions.

Abstract: In this talk, I explain how to study the local regularity and the convergence of some lacunary Fourier series. I will develop two approaches: one is based on the study of the Gauss dynamical system, the other one relies more on Fourier analysis. In both cases we will emphasize the key role played by Diophantine approximation.

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Speaker: Dalibor Volný, Université de Rouen, France.

Title: Martingale approximation and limit theorems in random fields

Abstract: When studying limit theorems for stationary random fields, martingale approximations have been considered difficult to use. Recently there have been several new results extending Maxwell-Woodroofe's condition and Hannan's condition to orthomartingales. We'll try to give their explication.