Fractal percolation is unrectifiable

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Résumé

This is a joint paper with Esa Järvenpää, Maarit Järvenpää, Tamás Keleti and Tuomas Pöyhtäri.

We show that there exists $0 < \alpha_0 < 1$ (depending on the parameters) such that the fractal (or Mandelbrot) percolation is almost surely purely α-unrectifiable for all $\alpha > \alpha_0$.

Given $0 < \alpha \leq 1$, a set$A \in \mathbb{R}^d$ is purely α-unrectifiable if $H^{1/\alpha}(A \cap \gamma([0,1])) = 0$ for all α-Hölder curves $\gamma : [0,1] \to \mathbb{R}^d$, where H^s is the s-dimensional Hausdorff measure.

We consider fractal percolation based on a subdivision of $[0,1]^d$ into N^d many subcubes.

It is verified that, for every $0 \leq p < 1$ and $N, d = 2, 3, ...$, there exists $\alpha_0 < 1$ such that almost surely the fractal percolation set E is purely α-unrectifiable for all $\alpha_0 < \alpha \leq 1$.

Since the case $\alpha = 1$ corresponds to standard 1-unrectifiability, this result implies that E is almost surely purely 1-unrectifiable.

In our paper a simpler proof than that of our main theorem is given for 1-unrectifiability. The general case, requiring new tools, is considered in Section 7 of the paper. We hope that these new tools turn out to be useful in many other problems related to the fractal percolation and other random geometric constructions.